## Section 11

# Geometric Area And The Pythagorean Theorem

**Step 1**
***Read Through The Entire Problem***

Determine which type of problem it is in order to know which formula to use to solve the problem. Is it a Geometric Area problem or is it a Pythagorean Theorem problem?

In a Geometric Area problem, the word "area" will be stated. In a Pythagorean Theorem problem, it will state "hypotenuse" and/or "right triangle". Look for Direct Translation words.
? ?

**Step 2**
***Name The Expressions***

In both Geometric Area and Pythagorean Theorem problems, you need to use Direct Translation to name the expressions you need for the formulas. As in previous sections, keep in mind you will need to name the Totally Unknown with a variable, and then use Direct Translation in order to build on that variable and name the other expressions.

In a Geometric Area problem, you will need to name expressions for both the length and the width of a rectangle.

In a Pythagorean Theorem problem, you will need to name expressions for the three sides of a right triangle. These three sides will generally be referred to as the shorter leg, the longer leg, and the hypotenuse.

**Step 3**
***Set Up An Equation By Substituting The Expressions Into The Appropriate Formula***

In Geometric Area, the value for the area is given; you need to name expressions for the length and width. In Pythagorean Theorem, you need to name expressions for all three sides.

| Problem Type | Geometric Shape | Formula |
|---|---|---|
| Geometric Area | Rectangle | $LW = A$ |
| Pythagorean Theorem | Right Triangle | $a^2 + b^2 = c^2$ |

***HELPFUL HINTS***

- With the Geometric Area formula, it may be easier to solve the equation if you set it up as $WL = A$, if the width ($W$) is the monomial (single term). $L \cdot W$ is the same as $W \cdot L$.
- When setting up the formula for the Pythagorean Theorem, it may be helpful to think of the formula as $(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$.

**Step 4**
***Solve the Equation***

Using the method taught by your instructor, solve the equation for the variable. Keep in mind when solving a Geometric Area or a Pythagorean Theorem Problem, you normally get two solutions to the equation. If one of the solutions is negative, eliminate it because the length of any side of a shape **cannot be negative**. If both of the solutions are positive, you must eliminate one of them by substituting each of them into the expressions named in Step 2. Whichever solution makes the value of an expression negative will be eliminated.

**Step 5**
***Make Sure to Answer the Question Being Asked***

As in previous sections, when you solve the equation you will find the value for $x$, but that might not be the answer to the question. You need to reread the problem and make sure exactly what question is being asked. It is possible that the value for the variable $x$ may be your answer. But it may *not* be.

For example, the value for $x$ may give you the *width* of a rectangle, while the problem may be asking for the *length* of the rectangle. You will have to substitute the value for $x$ into the original expression for the length that you will set up in Step 2 in order to get the correct answer. Always be sure of exactly what the question is.

## EXAMPLES

**EXAMPLE 1** The length of a rectangle is 8 feet shorter than three times its width. If the area of the rectangle is 35 square feet, what is the length of the rectangle?

**SOLUTION**

Step 1 *Read The Problem*

- Seeing the word "area" verifies that it is a Geometric Area problem.
- The problem contains Direct Translation Words [is, shorter than, three times ].

Step 2 *Name The Expressions*

- You are given information about the length; it is 8 feet shorter than three times the width.
- You are not given information about the width, so the width is the Totally Unknown.
- Use $x$ for the width. Use Direct Translation to name an expression for the length.
- The area is always given. It is 35.

$$\text{Width } (W) = x$$
$$\text{Length } (L) = 3x - 8$$

Step 3 *Set Up The Equation*

- The width is the monomial, the single term.
- As per Step 3 Helpful Hint, use the formula $WL = A$ instead of $LW = A$.
- The area is 35, so substitute the number 35 for $A$.
- Substitute the expressions you named in Step 2 for $L$ and $W$ into the formula.

$$x(3x - 8) = 35$$

Step 4 *Solve The Equation*

$$x = -\frac{7}{3}, x = 5$$

- The solutions to the equation are
- One of the solutions is negative.

- Eliminate the negative solution because the length of a side of a shape cannot be negative.
- The correct solution is $\boxed{x = 5}$

Step 5 *Answer The Question Asked*

- You have your solution to the equation, but it is NOT the answer to the question.
- The value of $x$ is the size of the width; the problem asks for the length.
- You need to use the expression for the length that you named in Step 2.
- Get the answer by substituting the solution for $x$ (which is 5) into the expression.

$$\begin{aligned} L &= 3x - 8 \\ L &= 3(5) - 8 \\ L &= 7 \end{aligned}$$

*Answer*: The length of the rectangle is 7 feet.

**EXAMPLE 2** The hypotenuse of a right triangle is one inch longer than the longer leg. The shorter leg is 7 inches shorter than the longer leg. Find the length of the hypotenuse.

**SOLUTION**

Step 1 *Read The Problem*

- Seeing the word "right triangle" verifies that it is a Pythagorean Theorem problem.
- The problem contains Direct Translation Words [is, longer than, shorter than ].

Step 2 *Name The Expressions*

- You are given information about the hypotenuse; it is 1 inch longer than the longer leg.
- You are given information about the shorter leg; it is 7 inches shorter than the longer leg.
- You are not given information about the longer leg, so that is the Totally Unknown.
- Use Direct Translation to name expressions for the hypotenuse and the shorter leg.

Step 3 *Set Up The Equation*

- Use the formula $a^2 + b^2 = c^2$ (Or $\text{Leg}^2 + \text{Leg}^2 = \text{Hypotenuse}^2$).
- Substitute the expressions you named for each leg and the hypotenuse into the formula.

$$x^2 + (x-7)^2 = (x+1)^2$$

Step 4 *Solve The Equation*

- The solutions to the equation are $x = 12,\ x = 5$
- When both solutions are positive, substitute each one into the expressions in Step 2.
- $x = 5$ is eliminated because it makes the expression for the Shorter Leg negative.
- The only solution you can use is $x = 12$

Step 5 *Answer The Question Asked*

- You have your solution to the equation, but it is NOT the answer to the question.
- The value of $x$ is the size of the longer leg; the problem asks for size of the hypotenuse.
- You need to use the expression for the hypotenuse you named in Step 2.
- Get the answer by substituting the solution for $x$ (which is 12) into the expression.

Hypotenuse = $x + 1$
Hypotenuse = $12 + 1$
Hypotenuse = 13

*Answer*: The length of the hypotenuse is 13 inches.

# Geometric Area and the Pythagorean Theorem: Exercise Set

**1.** The width of a rectangle is 7 inches shorter than its length. The area of the rectangle is 44 square inches. Find the length and width of the rectangle.

**2.** The width of a rectangle is 2 yards shorter than its length. The area of the rectangle is 48 square yards. Find the length and width of the rectangle.

**3.** The length of a rectangle is 5 feet longer than its width. The area of the rectangle is 66 square feet. Find the length of the rectangle.

**4.** The length of a rectangle is 4 inches longer than its width. The area of the rectangle is 96 square inches. Find the width of the rectangle.

**5.** The length of a rectangular picture frame is twice its width. If the area of the frame is 50 square inches, what are the dimensions of the picture frame?

**6.** The length of a rectangular rug is 3 feet shorter than twice its width. The area of the rug is 54 square feet. What are the dimensions of the rug?

**7.** The area of a rectangular pool cover is 15 square yards. If the length of the pool cover is one yard shorter than twice its width, what are the dimensions of the pool cover?

**8.** The area of a rectangular tabletop is 8 square feet. If the length of the tabletop is twice its width, what are the dimensions of the tabletop?

**9.** The length of a rectangle is 2 feet longer than twice its width. If the area of the rectangle is 40 square feet, what is the length of the rectangle?

**10.** The length of a rectangle is 3 inches longer than 3 times its width. If the area of the rectangle is 36 square inches, what is the length of the rectangle?

**11.** The longer leg of a right triangle is 7 feet longer than the shorter leg. The hypotenuse is 13 feet. Write the length of all three sides.

**12.** The longer leg of a right triangle is 7 feet longer than the shorter leg. The hypotenuse is 17 feet. Write the length of all three sides.

**13.** The shorter leg of a right triangle is 2 inches shorter than the longer leg. The hypotenuse is 10 inches. How long is the shorter leg?

**14.** The shorter leg of a right triangle is 14 yards shorter than the longer leg. The hypotenuse is 26 yards. How long is the shorter leg?

**15.** The hypotenuse of a right triangle is 4 meters longer than the longer leg. The shorter leg is 4 meters shorter than the longer leg. Find all three sides.

**16.** The longer leg of a right triangle is 2 feet shorter than twice the shorter leg. The hypotenuse is 2 feet longer than twice the shorter leg. How long is the hypotenuse?

**17.** The hypotenuse of a right triangle is 4 yards longer than 3 times the shorter leg. The longer leg is 3 yards longer than three times the shorter leg. Find the length of all three sides.

**18.** The hypotenuse is 5 inches shorter than 6 times the shorter leg. The longer leg is 5 inches longer than 5 times the shorter leg. Find the length of all three sides.

**19.** A 10 foot ladder is leaning against the side of a building. The distance from the bottom of the building to the top of the ladder is 2 more feet than the distance from the bottom of the building to the base of the ladder. What is the distance from the bottom of the building to the base of the ladder? (Hint: It would be helpful to draw a sketch.)

**20.** Two cars left the same intersection. One car traveled to the north. The other car traveled to the east. The car that went east drove 7 miles further than the car that traveled to the north. At that point the two cars were 13 miles away from each other. How far did each car travel? (Hint: It would be helpful to draw a sketch.)